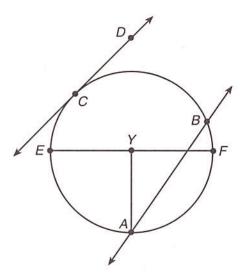
Lesson 6: Circles

In this lesson, you will review the parts of a circle. You will also apply the relationships between these parts to solve problems, such as finding the area of a sector and the lengths of arcs and line segments.

Parts of a Circle

A **circle** consists of all points in a plane that are an equal distance from a given point, called the center. The **center point** names the circle. Here is circle *Y*:



Radius (r): a line segment from the center point to any point on the circle. All radii (plural of radius) are equal in length. (\overline{YA} is a radius of circle Y.)

Diameter (d): a line segment that passes through the center and has both endpoints on the circle. All diameters are equal in length. (\overline{EF} is a diameter of circle Y.)

Chord: a line segment that has both endpoints lying on the circle. (\overline{AB} and \overline{EF} are chords of circle Y.)

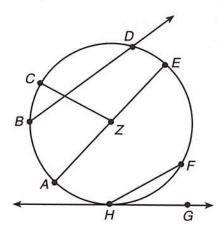
Arc: a portion of the circle that includes two endpoints and all the points in between. $\widehat{(CB)}$ and \widehat{CFA} are arcs of circle Y.)

Secant: a line, ray, or segment that contains a chord. (\overrightarrow{AB}) is a secant of circle Y.)

Tangent: a line, ray, or segment that intersects the circle at exactly one point. (\overrightarrow{CD}) is a tangent to circle Y. C is the **point of tangency** on circle Y.



Directions: Use the following circle to list the parts of the circle given in questions 1 through 8.



- 1. Z_____
- 2. *EF* ______
- 3. *CZ* _____
- 4. *HF* _____
- 5. H_____
- 6. HG _____
- 7. AE _____
- 8. \overrightarrow{BD} _____
- 9. Which statement is true?
 - A. A diameter is never a secant.
 - B. A diameter is always a chord.
 - C. A secant is sometimes a tangent.
 - D. A point of tangency is sometimes the center point.

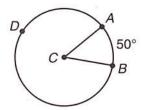
Central Angles

In a circle, a **central angle** is an angle with a vertex at the center of the circle. The segments of the angle are two radii of the circle. A central angle always makes a major and a minor arc. A **minor arc** has the same measure as the central angle, and its measure is always less than 180°. A **major arc** is found by subtracting the minor arc from 360°, and its measure is always more than 180°. Minor arcs are labeled with two letters, and major arcs are labeled with three letters.



Example

What is $m \angle ACB$ in circle C?



The measure of the minor arc is equal to the measure of the central angle.

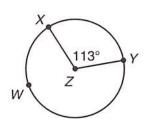
$$m \angle ACB = \widehat{mAB}$$

= 50°



Example

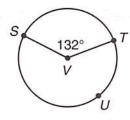
What is \widehat{mXWY} in circle Z?



Because
$$m \angle XZY = 113^\circ$$
, $mXY = 113^\circ$.
 $mXWY = 360^\circ - mXY$
 $= 360^\circ - 113^\circ$
 $= 247^\circ$

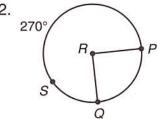
Directions: For questions 1 through 4, find the missing values.

1.



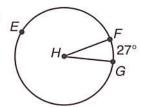
$$\widehat{mST} = \underline{\qquad}$$

$$\widehat{mSUT} =$$

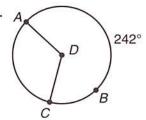


 $\widehat{mPSQ} = 270^{\circ}$

$$\widehat{mPQ} = \underline{\hspace{1cm}}$$



$$\widehat{mFEG} = \underline{\hspace{1cm}}$$



$$\widehat{mABC} = 242^{\circ}$$

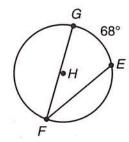
$$\widehat{mAC} = \underline{\qquad}$$

Inscribed Angles

In a circle, an **inscribed angle** is an angle with a vertex on the circle. The segments of the angle are two chords of the circle. The arc between the endpoints of those chords is called the **intercepted arc**. The measure of an inscribed angle is equal to half the measure of its intercepted arc.

Example

What is $m \angle EFG$ in circle H?



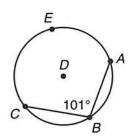
The measure of an inscribed angle is equal to half the measure of its intercepted arc.

$$m \angle EFG = \frac{1}{2}m\widehat{EG}$$

= $\frac{1}{2}(68^{\circ})$
= 34°

Example

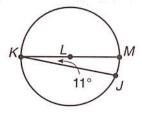
What is \widehat{mAC} in circle D?



Because
$$m \angle ABC = 101^\circ$$
, $\widehat{mAEC} = 202^\circ$.
 $\widehat{mAC} = 360^\circ - \widehat{mAEC}$
 $= 360^\circ - 202^\circ$
 $= 158^\circ$

Directions: For questions 1 through 4, find the missing values.

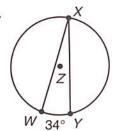
1.



$$\widehat{mJM} = \underline{\qquad}$$

$$\widehat{mJKM} =$$

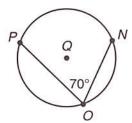
2.



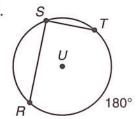
$$m \angle WXY =$$

$$\widehat{mWXY} = \underline{\hspace{1cm}}$$

3.



$$\widehat{NOP} = \underline{\hspace{1cm}}$$



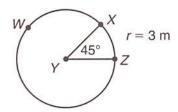
Finding the Length of a Circular Arc

If you know the circumference of a circle, you can use the measures of angles and arcs to find the lengths of minor, major, and intercepted arcs. You can solve such problems by using proportions or a formula.

>

Example

What is the length of \widehat{XZ} ? Use 3.14 for π .



Find the circumference of the circle.

$$C = 2\pi r = 2 \cdot 3.14 \cdot 3 = 18.84 \text{ m}$$

Set up a proportion and solve.

$$\frac{m\hat{X}Z}{18.84} = \frac{45}{360}$$

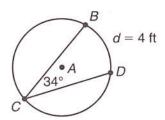
$$\widehat{mXZ} = 18.84 \cdot \frac{45}{360} = 2.355$$

The length of \widehat{XZ} is 2.355 m.



Example

What is the length of \widehat{BD} ? Use 3.14 for π .



Substitute the known values into the following formula and solve.

arc length =
$$2\pi r \cdot \left(\frac{\text{arc in degrees}}{360^{\circ}}\right)$$

= $2 \cdot 3.14 \cdot 2 \cdot \left(\frac{68}{360}\right) = 2.3724444444...$

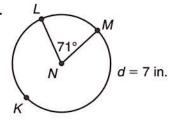
The length of \widehat{BD} is about 2.37 ft.

0

Practice

Directions: For questions 1 through 4, find the missing values. Use 3.14 for π , and round your answer to the nearest hundredth.

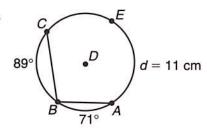
1.



$$\widehat{mLM} = \underline{\qquad}$$

length of
$$\widehat{LM} \approx$$

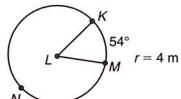
2.



$$\widehat{mAEC} =$$

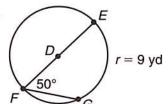
length of
$$\widehat{AEC}$$
 ≈ _____

3.



$$\widehat{mKNM} = \underline{\hspace{1cm}}$$

length of
$$\widehat{KNM} \approx$$



length of
$$\widehat{EFG} \approx$$

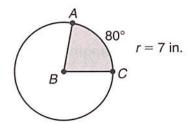
Finding the Area of a Sector

A **sector** is a region of a circle that is bounded by two radii and an arc. You can find the area of a sector by using proportions or a formula.

>

Example

What is the area of the shaded sector? Use 3.14 for π .



Find the area of the circle.

$$A = \pi r^2 = 3.14 \cdot 7^2 = 153.86 \text{ in.}^2$$

Set up a proportion and solve.

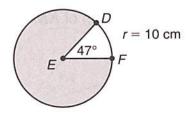
$$\frac{x}{153.86} = \frac{80}{360}$$
$$x = 153.86 \cdot \frac{80}{360} = 34.19111111...$$

The area of the shaded sector is about 34.19 in.²



Example

What is the area of the shaded sector? Use 3.14 for π .



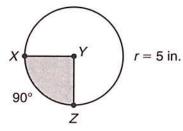
Substitute the known values into the following formula to solve.

$$A = \pi r^2 \bullet \left(\frac{\text{degrees in corresponding arc}}{360^\circ}\right)$$
$$= 3.14 \bullet 10^2 \bullet \left(\frac{313}{360}\right) = 273.0055555...$$

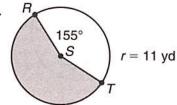
The area of the shaded sector is about 273.01 cm².

Directions: For questions 1 through 4, find the area of the shaded sector. Use 3.14 for π , and round your answer to the nearest hundredth.

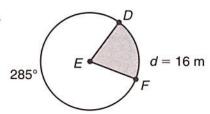
1.



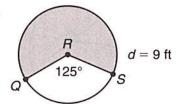
2.



3.



A≈

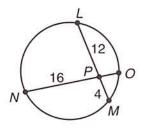


Finding the Lengths of Chord Segments

When two chords intersect, they form four segments. The product of the lengths of the two segments of one chord always equals the product of the lengths of the two segments of the other chord. If the length of one or more of the segments is unknown, you can set up an equation and solve for the unknown.

Example

Find the length of \overline{PO} .



$$NP \cdot PO = LP \cdot PM$$

 $16 \cdot x = 12 \cdot 4$

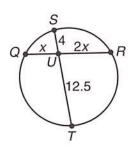
$$x = 3$$

The length of \overline{PO} is 3.



Example

Find the lengths of \overline{QU} and \overline{UR} .



$$QU \bullet UR = SU \bullet UT$$

$$x \cdot 2x = 4 \cdot 12.5$$

$$2x^2 = 50$$

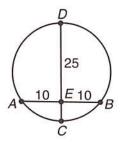
$$x^2 = 25$$

$$x = 5$$

The length of \overline{QU} is 5, and the length of \overline{UR} is 10.

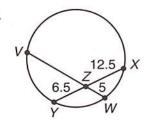
Directions: For questions 1 through 4, find the missing lengths.

1.



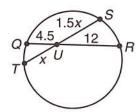
EC = _____

2.



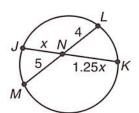
VZ = _____

3.



TU = _____

4.



JN = _____

Finding Angle Measures Formed by Chords, Secants, and Tangents

If two chords intersect inside a circle, the measure of each angle is half the sum of the measures of the arcs intercepted by the angle and its vertical angle.

| Figure | Formula |
|-------------|---|
| Chord-Chord | |
| | $m \angle x = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$ |

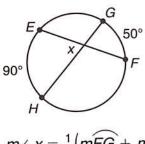
If two secants, two tangents, or a secant and a tangent intersect outside of a circle, then the measure of the angle formed by that intersection is half the difference of the measures of the intercepted arcs.

| Figure | Formula |
|------------------------|--|
| Tangent–Secant | $m \angle x = \frac{1}{2}(m\widehat{ADC} - m\widehat{BC})$ |
| Tangent–Tangent | $m \angle x = \frac{1}{2}(m\widehat{ACB} - m\widehat{AB})$ |
| Secant–Secant C A D | $m \angle x = \frac{1}{2} (m\widehat{CD} - m\widehat{AB})$ |



Example

What is the value of x?

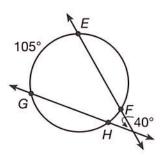


$$m \angle x = \frac{1}{2} \left(\widehat{mFG} + \widehat{mEH} \right)$$
$$= \frac{1}{2} (50^{\circ} + 90^{\circ})$$
$$= 70^{\circ}$$

E

Example

What is the value of \widehat{mFH} ?



$$\frac{1}{2}(\widehat{mGE} - \widehat{mFH}) = 40^{\circ}$$

$$\frac{1}{2}(105^{\circ} - m\widehat{FH}) = 40^{\circ}$$

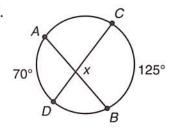
$$(105^{\circ} - m\widehat{FH}) = 80^{\circ}$$

$$-m\widehat{FH} = -25^{\circ}$$

$$\widehat{mFH} = 25^{\circ}$$

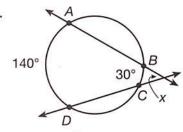
Directions: For questions 1 through 4, find the value of x.

1.

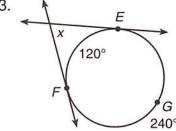


x = _____

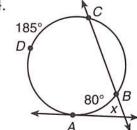
2.



x = _____

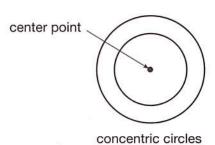


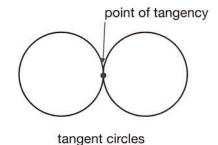
x = _____



Applications of Circles

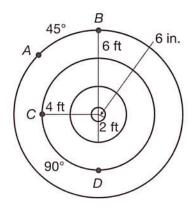
Circles that have a common center point are concentric circles. Circles that share a common point of tangency are tangent circles.





Example

Curling is a winter sport in which players slide stones on a sheet of ice toward a target. The target is a series of concentric circles. The figure below shows the location of four stones on the target area. What is the difference between the arc length between stones A and B, AB, and the arc length between stones C and D, CD? Use 3.14 for π .



Find the circumference of the circle with stones A and B and the circle with stones C and D.

Circle with radius of 6 ft

Circle with radius of 4 ft

$$C = 2\pi r = 2 \times 3.14 \times 6 \text{ ft} = 37.68 \text{ ft}$$

$$C = 2\pi r = 2 \times 3.14 \times 4 \text{ ft} = 25.12 \text{ ft}$$

Set up two proportions and solve.

$$\frac{\widehat{mAB}}{37.68} = \frac{45}{360}$$

$$\frac{\widehat{mCD}}{25,12} = \frac{90}{360}$$

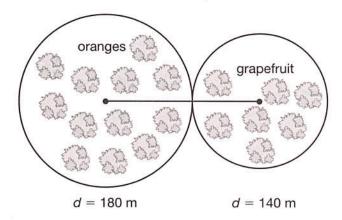
$$\widehat{mAB} = 37.68 \times \frac{45}{360} = 4.71$$
 $\widehat{mCD} = 25.12 \times \frac{90}{360} = 6.28$

$$\widehat{mCD} = 25.12 \times \frac{90}{360} = 6.28$$

The length of AB is 4.71 ft. The length of CD is 6.28 ft. The difference is 6.28 - 4.71 = 1.57 feet.

Example

A farmer uses tangent circles to plant his citrus fruit trees. The circular plot for his orange trees has a diameter of 180 meters. The circular plot for his grapefruit trees has a diameter of 140 meters.



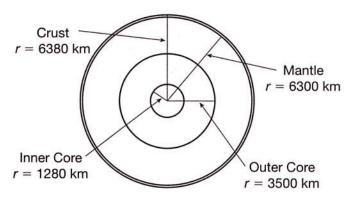
What is the distance between the centers of the two circular plots?

The diameter of the orange plot is 180 meters, so its radius is 90 meters. The diameter of the grapefruit plot is 140 meters, so its radius is 70 meters. The tangent circles share a point of tangency, so their radii are connected. The distance from one center to the other is the sum of the two radii.

$$90 \text{ m} + 70 \text{ m} = 160 \text{ m}$$

Directions: Use the cross-section of Earth to answer questions 1 through 4. Use 3.14 for π . These values are approximate.

Cross-Section of Planet Earth



1. What is the diameter of planet Earth?

- 2. What is the area of the outer core on the cross-section of Earth? Be sure not to include the area of the inner core. Round your answer to the nearest km².
- 3. The northern hemisphere represents half of Earth. What is the arc length of the outer edge of the cross-section of the mantle of the northern hemisphere?
- 4. What is the area of the crust on the cross-section of Earth? Be sure not to include the areas of the parts beneath the crust. Round your answer to the nearest km².